

Løsningsforslag til 9. utgave

Matematikk for økonomi og samfunnsfag

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Grunnleggende emner

OPPGAVE 1.1 Vi skal trekke sammen uttrykkene

- a) $7a + 3b - 5b + 4c - 3a + 6b + 5ac = 4a + 4b + 4c + 5ac$
- b) $2x^2 - 3x^3 + 4x + 10x^2 - 3x = -3x^3 + 12x^2 + x$
- c) $5xy + 4xz - 3yx + 3yz - xy + xz = xy + 5xz + 3yz$
- d) $A^2 + AB - aB + 4AB - 3A^2 = -2A^2 + 5AB - aB$

OPPGAVE 1.2

- a) $x + y + (x + y) = x + y + x + y = 2x + 2y$
- b) $x + y + (x - y) = x + y + x - y = 2x$
- c) $x + y + (-x + y) = x + y - x + y = 2y$
- d) $x - y - (x + y) = x - y - x - y = -2y$
- e) $x - y - (x - y) = x - y - x + y = 0$
- f) $x - y - (-x - y) = x - y + x + y = 2x$

OPPGAVE 1.3 Vi har at $x = 3$, $y = -2$, $z = -1$

- a) $5x - 3y + 2z = 5 \cdot 3 - 3 \cdot (-2) + 2 \cdot (-1) = 15 + 6 - 2 = 19$
- b) $2x - y^2 + 3z^3 = 2 \cdot 3 - (-2)^2 + 3 \cdot (-1)^3 = 6 - 4 + 3 \cdot (-1) = 6 - 4 - 3 = -1$
- c) $xy - xz - (x - yz) = 3 \cdot (-2) - 3 \cdot (-1) - (3 - (-2) \cdot (-1)) = -6 + 3 - (3 - 2) = -6 + 3 - 1 = -7 + 3 = -4$
- d) $x^2 + y^2 + z^2 - 2x - 3y - 4z - (x^2 - 2xz + 3xy) = 3^2 + (-2)^2 + (-1)^2 - 2 \cdot 3 - 3 \cdot (-2) - 4 \cdot (-1) - (3^2 - 2 \cdot 3 \cdot (-1) + 3 \cdot 3 \cdot (-2)) = 9 + 4 + 1 - 6 + 6 + 4 - (9 + 6 - 18) = 18 - (-3) = 18 + 3 = 21$

OPPGAVE 1.4

- a) $a(a^2 + 3a + 4) - a^2(a - 5)$
 $= a^3 + 3a^2 + 4a - a^3 + 5a^2 = 8a^2 + 4a$
- b) $(2x - 1)(1 - x) = 2x - 1 - 2x^2 + x = -2x^2 + 3x - 1$
- c) $(y^2 - x^2)(y - x) - (x^2 - y^2)(x - y)$
 $= y^3 - x^2y - xy^2 + x^3 - (x^3 - xy^2 - x^2y + y^3)$
 $= y^3 - x^2y - xy^2 + x^3 - x^3 + xy^2 + x^2y - y^3$
 $= 0$

OPPGAVE 1.5

- a) $2abc = 2 \cdot (-1) \cdot 2 \cdot (-2) = 8$
- b) $4a^3b^2c^5 = 4(-1)^3 \cdot 2^2 \cdot (-2)^5 = 4 \cdot (-1) \cdot 4 \cdot (-32)$
 $= 512$
- c) $(8b - a)(c - 3b) = [8 \cdot 2 - (-1)][(-2) - 3 \cdot 2]$
 $= (16 + 1)(-2 - 6) = 17 \cdot (-8) = -136$

OPPGAVE 1.6

Vi skal regne ut uttrykkene

y_1, y_2 og y_3 for $x = -2, -1, 0, 1$ og 2

$$\begin{aligned}y_1 &= 2x + 1 \\y_2 &= -3x + 4 \\y_2 &= -2x^2 + 3x - 4\end{aligned}$$

og ser først på y_1 :

$$\begin{aligned}x = -2 & \quad y_1 = 2(-2) + 1 = -4 + 1 = -3 \\x = -1 & \quad y_1 = 2(-1) + 1 = -2 + 1 = -1 \\x = 0 & \quad y_1 = 1 \\x = 1 & \quad y_1 = 2 \cdot 1 + 1 = 2 + 1 = 3 \\x = 2 & \quad y_1 = 2 \cdot 2 + 1 = 4 + 1 = 5\end{aligned}$$

For y_2 får vi disse verdiene:

$$\begin{aligned}x = -2 & \quad y_2 = (-3)(-2) + 4 = 6 + 4 = 10 \\x = -1 & \quad y_2 = (-3)(-1) + 4 = 3 + 4 = 7 \\x = 0 & \quad y_2 = 4 \\x = 1 & \quad y_2 = (-3) \cdot 1 + 4 = -3 + 4 = 1 \\x = 2 & \quad y_2 = (-3) \cdot 2 + 4 = -6 + 4 = -2\end{aligned}$$

Så regner vi ut verdiene for y_3 :

$$\begin{aligned}x = -2 & \quad y_3 = -2(-2)^2 + 3(-2) - 4 = -2 \cdot 4 - 6 - 4 \\& \quad = -8 - 6 - 4 = -18 \\x = -1 & \quad y_3 = -2(-1)^2 + 3(-1) - 4 = -2 \cdot 1 - 3 - 4 \\& \quad = -2 - 3 - 4 = -9 \\x = 0 & \quad y_3 = -2(0)^2 + 3(0) - 4 = -4 \\x = 1 & \quad y_3 = -2(1)^2 + 3(1) - 4 = -2 \cdot 1 + 3 - 4 \\& \quad = -2 + 3 - 4 = -3 \\x = 2 & \quad y_3 = -2(2)^2 + 3(2) - 4 = -2 \cdot 4 + 6 - 4 \\& \quad = -8 + 6 - 4 = -6\end{aligned}$$

Dette oppsummerer vi i tabellen:

x	$y_1 = 2x + 1$	$y_2 = -3x + 4$	$y_3 = -2x^2 + 3x - 4$
-2	-3	10	-18
-1	-1	7	-9
0	1	4	-4
1	3	1	-3
2	5	-2	-6

OPPGAVE 1.7

- a) $5^2 \cdot 5 \cdot 5^3 = 5^{2+1+3} = 5^6 = 15625$
 b) $x \cdot x^2 \cdot x^3 = x^{1+2+3} = x^6$
 c) $y + y^2 + y + y^2 = 2y + 2y^2$
 d) $\frac{a \cdot b^3}{b^2} = ab^{3-2} = ab^1 = ab$
 e) $\frac{9F^3 \cdot 2G^2 \cdot 5H^4}{6FGH} = \frac{9 \cdot 2 \cdot 5}{6} F^{3-1} G^{2-1} H^{4-1} = 15F^2GH^3$
 f) $\frac{5abc}{20abc} = \frac{1}{4}$

OPPGAVE 1.8

- a) $3^{-1} \cdot 3^0 \cdot 3^2 = 3^{-1+0+2} = 3^1 = 3$
 b) $2^4 \cdot X^2 \cdot 2^{-2} \cdot X^{-2} \cdot 2^{-3} = 2^{4-2-3} X^{2-2} = 2^{-1} X^0 = \frac{1}{2}$
 c) $\frac{a^5 c^{-3}}{c} b^3 c^4 = a^5 b^3 c^{-3+4-1} = a^5 b^3 c^0 = a^5 b^3$
 d) $\frac{24s^3 t^3 u^{10} t^{-2} a}{6t^4 u^5 3a^2} = \frac{24}{6 \cdot 3} a^{1-2} s^3 t^{3-2-4} u^{10-5} = \frac{4}{3} a^{-1} s^3 t^{-3} u^5$
 $= \frac{4s^3 u^5}{3at^3}$

OPPGAVE 1.9

Når $a = -3$, $b = 2$ og $c = \frac{1}{2}$, får vi at

$$\text{a) } 2a^2 \cdot \frac{c^3}{b^4} = \frac{2 \cdot (-3)^2 \cdot \left(\frac{1}{2}\right)^3}{2^4} = \frac{2 \cdot 9 \cdot \frac{1}{8}}{16} = \frac{2 \cdot 9}{16 \cdot 8} = \frac{9}{64}$$

$$\begin{aligned}
 \text{b)} \quad \frac{3^4 b^{-3}}{5a^{-2}} \cdot \frac{5^2 b^2}{3a^3} &= \frac{3^4 \cdot 2^{-3} \cdot 5^2 \cdot 2^2}{5 \cdot (-3)^{-2} \cdot 3 \cdot (-3)^3} \\
 &= -2^{-3+2} 3^{4+2-1-3} 5^{2-1} \\
 &= -2^{-1} 3^2 5^1 = -\frac{9 \cdot 5}{2} = -\frac{45}{2}
 \end{aligned}$$

OPPGAVE 1.10 Vi skal regne ut

$$\begin{aligned}
 \text{a)} \quad (6x^4)^3 \cdot \left(\frac{1}{3}x^2\right)^2 &= \frac{6^3(x^4)^3 \cdot (x^2)^2}{3^2} = \frac{2^3 3^3 x^{12} x^4}{3^2} \\
 &= 8 \cdot 3x^{12+4} = 24x^{16} \\
 \text{b)} \quad (2a^4 b^3 c^{-2})^2 \cdot (4c^3 a^2 b^4)^4 &= 2^2 a^8 b^6 c^{-4} 4^4 c^{12} a^8 b^{16} \\
 &= 1024 a^{8+8} b^{6+16} c^{-4+12} \\
 &= 1024 a^{16} b^{22} c^8 \\
 \text{c)} \quad \frac{d^{2n+5}}{d^{n-1}} &= d^{(2n+5)-(n-1)} = d^{2n+5-n+1} = d^{n+6} \\
 \text{d)} \quad \frac{d^{n-1}}{d^{2n+5}} &= d^{n-1-(2n+5)} = d^{n-1-2n-5} = d^{-n-6} = \frac{1}{d^{n+6}} \\
 \text{e)} \quad \frac{5e^{-x+3}e^{3x+1}}{e^{2-x}e^{4x-2}} &= 5e^{-x+3+(3x+1)-(2-x)-(4x-2)} \\
 &= 5e^{-x+3+3x+1-2+x-4x+2} \\
 &= 5e^{-x+4}
 \end{aligned}$$

OPPGAVE 1.11 Vi skal forenkle potensene

$$\begin{aligned}
 \text{a)} \quad x^4 x^{-3} x^2 &= x^{4-3+2} = x^3 \\
 \text{b)} \quad \frac{(2a)^3 a^{-1}}{a^3 a} &= 2^3 a^3 a^{-1-3-1} = 8a^{3-1-3-1} = 8a^{-2} = \frac{8}{a^2} \\
 \text{c)} \quad \frac{\left(\frac{x}{y}\right)^n \cdot \left(\frac{y}{x}\right)^{n+1}}{4y(x^2)^{-1}} &= \frac{x^n y^{n+1}}{4yx^{-2}y^n x^{n+1}} = \frac{1}{4} x^{n+2-(n+1)} y^{n+1-1-n} \\
 &= \frac{1}{4} x^{n+2-n-1} y^{n+1-1-n} = \frac{1}{4} x^1 y^0 = \frac{1}{4} x
 \end{aligned}$$

OPPGAVE 1.12 a) -6 b) 12 c) -100 d) 24

OPPGAVE 1.13 Skriv på standardform:

$$12300 = 1,23 \cdot 10^4 \qquad 345,7 = 3,457 \cdot 10^2$$

$$0,143 = 1,43 \cdot 10^{-1} \qquad 0,000065 = 6,5 \cdot 10^{-5}$$

$$0,00000011 = 1,1 \cdot 10^{-7}$$

Skriv på vanlig form:

$$2,0 \cdot 10^3 = 2000 \qquad 7,8 \cdot 10^8 = 780\,000\,000$$

$$1,25 \cdot 10^{-2} = 0,0125 \qquad 3,0 \cdot 10^{-5} = 0,000\,030$$

OPPGAVE 1.14 a) $3,15 \cdot 10^7$ s b) $4,0 \cdot 10^{13}$ km**OPPGAVE 1.15** a) $\sqrt{900} = \sqrt{30 \cdot 30} = 30$

b) $\sqrt{1000000} = \sqrt{1000 \cdot 1000} = 1000$

c) $\sqrt{\frac{25}{100}} = \sqrt{\frac{5 \cdot 5}{10 \cdot 10}} = \frac{5}{10} = \frac{1}{2}$

d) $\sqrt{25 - 9} = \sqrt{16} = \sqrt{4 \cdot 4} = 4$ NB!

e) $\sqrt{0,004}$ Kan ikke gjøre noen pen forenkling ($\approx 0,0632$)

f) $\sqrt{56,25} = \sqrt{7,5 \cdot 7,5} = 7,5$

g) $\sqrt{100 + 25} = \sqrt{125} = \sqrt{25 \cdot 5} = \sqrt{25} \cdot \sqrt{5} = 5 \cdot \sqrt{5}$

OPPGAVE 1.16 a) 10 b) $6\sqrt{5} - 4\sqrt{2}$ c) $3\sqrt{3}$ d) $\frac{1}{2}$ e) 6**OPPGAVE 1.17** Vi skal gjøre disse uttrykkene enklere og eventuelle nevner rasjonale.

a) $5\sqrt{3} \cdot 2\sqrt{6} = 5 \cdot 2\sqrt{3 \cdot 6} = 10\sqrt{3 \cdot 3 \cdot 2} = 10 \cdot 3\sqrt{2} = 30\sqrt{2}$

b) $(\sqrt{5})^5 = \sqrt{5^5} = \sqrt{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = 5 \cdot 5 \cdot \sqrt{5} = 25\sqrt{5}$

c) $\frac{15}{\sqrt{5}} = \frac{15 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{15\sqrt{5}}{5} = 3\sqrt{5}$

d) $\sqrt{2} \cdot \sqrt{3} \cdot \sqrt{12} = \sqrt{2 \cdot 3 \cdot 2 \cdot 2 \cdot 3} = 2 \cdot 3 \cdot \sqrt{2} = 6\sqrt{2}$

e) $\frac{30x}{\sqrt{30x}} = \frac{30x \cdot \sqrt{30x}}{\sqrt{30x} \cdot \sqrt{30x}} = \frac{30x \cdot \sqrt{30x}}{30x} = \sqrt{30x}$

f) $\frac{\sqrt{a^2 + b^2}}{\sqrt{a + b}} = \frac{\sqrt{a + b} \sqrt{a^2 + b^2}}{\sqrt{a + b} \cdot \sqrt{a + b}} = \frac{\sqrt{(a + b)(a^2 + b^2)}}{a + b}$

g) $3\sqrt{6} \cdot 2\sqrt{10} = 6 \cdot \sqrt{2 \cdot 3 \cdot 2 \cdot 5} = 6 \cdot 2 \cdot \sqrt{15} = 12\sqrt{15}$

$$\begin{aligned}
\text{h)} \quad & 3\sqrt{8} + 4\sqrt{18} - \sqrt{50} = 3\sqrt{2 \cdot 2 \cdot 2} + 4\sqrt{3 \cdot 3 \cdot 2} - \sqrt{5 \cdot 5 \cdot 2} \\
& = 3 \cdot 2\sqrt{2} + 4 \cdot 3 \cdot \sqrt{2} - 5\sqrt{2} \\
& = 6\sqrt{2} + 12\sqrt{2} - 5\sqrt{2} = 13\sqrt{2} \\
\text{i)} \quad & \sqrt{5} \cdot \sqrt{14} \cdot \sqrt{35} = \sqrt{5 \cdot 2 \cdot 7 \cdot 5 \cdot 7} = 5 \cdot 7\sqrt{2} = 35\sqrt{2} \\
\text{j)} \quad & 3\sqrt{18} - 4\sqrt{8} + 2\sqrt{50} = 3\sqrt{3 \cdot 3 \cdot 2} - 4\sqrt{2 \cdot 2 \cdot 2} + 2\sqrt{5 \cdot 5 \cdot 2} \\
& = 3 \cdot 3\sqrt{2} - 4 \cdot 2\sqrt{2} + 2 \cdot 5\sqrt{2} = 9\sqrt{2} - 8\sqrt{2} + 10\sqrt{2} \\
& = 11\sqrt{2} \\
\text{k)} \quad & \frac{\sqrt{p} + 1}{\sqrt{p}} = \frac{(\sqrt{p} + 1) \cdot \sqrt{p}}{\sqrt{p} \cdot \sqrt{p}} = \frac{\sqrt{p} \cdot \sqrt{p} + \sqrt{p}}{p} = \frac{p + \sqrt{p}}{p} \\
\text{l)} \quad & \frac{\sqrt{a} \cdot \sqrt{b^3c}}{(\sqrt{abc})^3} = \frac{\sqrt{a \cdot b \cdot b \cdot b \cdot c}}{(\sqrt{abc})^3} = \frac{b\sqrt{abc}}{(\sqrt{abc})^3} = \frac{b}{(\sqrt{abc})^2} \\
& = \frac{b}{abc} = \frac{1}{ac}
\end{aligned}$$

OPPGAVE 1.18

$$\begin{array}{ll}
\text{a)} \quad 9x^2 + 30xy + 25y^2 & \text{b)} \quad 9x^2 - 30xy + 25y^2 \\
\text{c)} \quad 9x^2 - 25y^2 & \text{d)} \quad x^4 - 2x^2 + 1 \\
\text{e)} \quad x^8 - y^8 & \text{f)} \quad 67r^2 - 111rs + 63s^2
\end{array}$$

OPPGAVE 1.19

Bruk kvadratsetningene og regn ut

$$\begin{aligned}
\text{a)} \quad & (3a - b)^2 = 9a^2 - 2 \cdot 3a \cdot b + b^2 = 9a^2 - 6ab + b^2 \\
\text{b)} \quad & \left(\frac{x}{2} + \frac{y}{3}\right)^2 = \left(\frac{x}{2}\right)^2 + 2 \cdot \frac{x}{2} \cdot \frac{y}{3} + \left(\frac{y}{3}\right)^2 = \frac{x^2}{4} + \frac{xy}{3} + \frac{y^2}{9} \\
\text{c)} \quad & (ab + ac)(ab - ac) = (ab)^2 - (ac)^2 = a^2b^2 - a^2c^2 \\
& = a^2(b^2 - c^2) \\
\text{d)} \quad & (-x - 2y)^2 = (-x)^2 + 2 \cdot (-x) \cdot (-2y) + (-2y)^2 \\
& = x^2 + 4xy + 4y^2
\end{aligned}$$

Faktoriser

$$\begin{aligned}
\text{e)} \quad & a^2 + 4ab + 4b^2 = a^2 + 2 \cdot a \cdot 2b + (2b)^2 = (a + 2b)^2 \\
\text{f)} \quad & 64z^2 - 81q^2 = (8z)^2 - (9q)^2 = (8z + 9q)(8z - 9q) \\
\text{g)} \quad & 0,16a^2 - c^2 = (0,4a)^2 - c^2 = (0,4a + c)(0,4a - c) \\
\text{h)} \quad & 0,5x^2 - 2x + 2 = \frac{1}{2}(x^2 - 4x + 4) = \frac{1}{2}(x - 2)^2 \\
\text{i)} \quad & 3t^3 - 75t^5 = 3t^3(1 - 25t^2) = 3t^3(1 + 5t)(1 - 5t)
\end{aligned}$$

OPPGAVE 1.20

$$\begin{aligned}
\text{a)} \quad & a^3 - 4a^2 + 4a = a(a - 2)^2 \\
\text{b)} \quad & x^2 - 9y^2 = (x + 3y)(x - 3y)
\end{aligned}$$

- c) $9p^2 - 25r^2 = (3p + 5r)(3p - 5r)$
 d) $4x^4 - 12ax^2 + 9a^2 = (2x^2 - 3a)^2$
 e) $\frac{1}{4}s^2 - 81t^2 = \left(\frac{1}{2}s + 9t\right)\left(\frac{1}{2}s - 9t\right)$
 f) $4XY^2 - 16XYZ + 16XZ^2 = 4X(Y - 2Z)^2$

OPPGAVE 1.21 Skriv brøkene som blandede tall.

- a) $\frac{25}{7} = 3\frac{4}{7}$ b) $\frac{145}{12} = \frac{144 + 1}{12} = 12\frac{1}{12}$
 c) $\frac{800}{5} = 160$ d) $\frac{110}{50} = \frac{11}{5} = \frac{10 + 1}{5} = 2\frac{1}{5}$

OPPGAVE 1.22 Skriv de blandede tallene som brøker.

- a) $7\frac{2}{7} = \frac{51}{7}$ b) $9\frac{3}{16} = \frac{147}{16}$
 c) $12\frac{5}{8} = \frac{101}{8}$ d) $75\frac{21}{34} = \frac{2571}{34}$

OPPGAVE 1.23 Forkort brøkene

- a) $\frac{100}{6} = \frac{\cancel{2} \cdot 2 \cdot 5 \cdot 5}{\cancel{2} \cdot 3} = \frac{50}{3}$ b) $\frac{x+y}{4x+4y} = \frac{\cancel{(x+y)}}{4\cancel{(x+y)}} = \frac{1}{4}$
 c) $\frac{3a}{3a+5ab} = \frac{\cancel{3}a}{\cancel{3}(3+5b)} = \frac{3}{3+5b}$
 d) $\frac{3a+3ab}{3b} = \frac{\cancel{3}a(1+b)}{\cancel{3}b} = \frac{a+ab}{b}$

OPPGAVE 1.24 Gjør nevnerne rasjonale i brøkene.

- a) $\frac{15}{\sqrt{5}} = 3\sqrt{5}$ b) $\frac{\sqrt{3}+3}{\sqrt{3}} = \sqrt{3} + 1$
 c) $\frac{\sqrt{2}+\sqrt{12}}{\sqrt{2}} = \sqrt{6} + 1$

OPPGAVE 1.25 Gjør nevnerne rasjonale.

- a) $\frac{2}{\sqrt{3}+\sqrt{2}} = \frac{2 \cdot (\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{2 \cdot (\sqrt{3}-\sqrt{2})}{3-2}$
 $= 2 \cdot (\sqrt{3}-\sqrt{2})$

$$\begin{aligned} \text{b) } \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} &= \frac{(\sqrt{5} - \sqrt{3})^2}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})} = \frac{5 - 2\sqrt{15} + 3}{5 - 3} \\ &= \frac{8 - 2\sqrt{15}}{2} = \frac{2(4 - \sqrt{15})}{2} = 4 - \sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{x\sqrt{x} + y\sqrt{y}}{\sqrt{x} - \sqrt{y}} &= \frac{(x\sqrt{x} + y\sqrt{y})(\sqrt{x} + \sqrt{y})}{(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})} \\ &= \frac{x\sqrt{x}\sqrt{x} + y\sqrt{y}\sqrt{y} + x\sqrt{x}\sqrt{y} + y\sqrt{y}\sqrt{x}}{x - y} \\ &= \frac{x^2 + y\sqrt{xy} + x\sqrt{xy} + y^2}{x - y} \\ &= \frac{x^2 + y^2 + (x + y)\sqrt{xy}}{x - y} \end{aligned}$$

OPPGAVE 1.26

$$\begin{aligned} \text{a) } \frac{7}{16} - \frac{5}{6} - \frac{11}{12} &= -\frac{21}{16} & \text{b) } 3 + \frac{5}{4} - \frac{7}{6} &= \frac{37}{12} \\ \text{c) } 1\frac{1}{3} + \frac{5}{7} + \frac{8}{35} &= \frac{239}{105} \end{aligned}$$

OPPGAVE 1.27 Gjør brøkuttrykkene enklere.

$$\begin{aligned} \text{a) } \frac{5}{2a} - \frac{3}{a} + 2 &= \frac{5}{2a} - \frac{3 \cdot 2}{a \cdot 2} + \frac{2 \cdot 2a}{2a} = \frac{5 - 6 + 4a}{2a} = \frac{4a - 1}{2a} \\ \text{b) } \frac{5x}{2} - \frac{36}{5} + \frac{1}{10} &= \frac{5x \cdot 5 - 36 \cdot 2 + 1}{10} = \frac{25x - 71}{10} \\ \text{c) } \frac{5}{12x} + \frac{3}{4x} - \frac{4}{3x} &= \frac{5 + 3 \cdot 3 - 4 \cdot 4}{12x} = \frac{-2}{12x} = -\frac{1}{6x} \end{aligned}$$

OPPGAVE 1.28

$$\begin{aligned} \text{a) } 0 & & \text{b) } \frac{10a}{3x} & & \text{c) } \frac{17x}{2x + y} \\ \text{d) } \frac{r^2 - 4}{2r^2 + 7r + 3} \end{aligned}$$

OPPGAVE 1.29 Trekk sammen brøkene.

$$\begin{aligned} \text{a) } \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{5}{6} &= \frac{1 \cdot 6}{2 \cdot 6} - \frac{1 \cdot 4}{3 \cdot 4} + \frac{1 \cdot 3}{4 \cdot 3} - \frac{5 \cdot 2}{6 \cdot 2} \\ &= \frac{6 - 4 + 3 - 10}{12} = \frac{-5}{12} \\ \text{b) } \frac{x}{5} + \frac{2x}{3} - \frac{7x}{45} &= \frac{x \cdot 9}{5 \cdot 9} + \frac{2x \cdot 15}{3 \cdot 15} - \frac{7x}{45} \\ &= \frac{9x + 30x - 7x}{45} = \frac{32x}{45} \end{aligned}$$

$$c) \frac{b}{a} + \frac{a}{b} = \frac{b \cdot b}{a \cdot b} + \frac{a \cdot a}{b \cdot a} = \frac{b^2 + a^2}{ab}$$

$$d) \frac{3}{ab} + \frac{4}{b^2} - \frac{1}{a^2b} = \frac{3 \cdot ab}{ab \cdot ab} + \frac{4 \cdot a^2}{b^2 \cdot a^2} - \frac{1 \cdot b}{a^2b \cdot b} \\ = \frac{3ab + 4a^2 - b}{a^2b^2} = \frac{4a^2 + 3ab - b}{a^2b^2}$$

$$e) \frac{2}{x+y} - \frac{2}{x-y} = \frac{2 \cdot (x-y)}{(x+y) \cdot (x-y)} - \frac{2 \cdot (x+y)}{(x-y) \cdot (x+y)} \\ = \frac{2(x-y) - 2(x+y)}{(x+y)(x-y)} = \frac{2x - 2y - 2x - 2y}{x^2 - y^2} \\ = \frac{-4y}{x^2 - y^2}$$

OPPGAVE 1.30 a) $\frac{-2}{x^2 - 1}$ b) $\frac{2a^2 + ab + b^2}{(a+b)(a-b)^2}$ c) $\frac{t}{t+1}$ d) $\frac{3(x-11)}{2(x-12)}$

OPPGAVE 1.31 Vi skal trekke sammen brøkene og forkorte om det er mulig.

$$a) \frac{x}{x-2} + \frac{3}{2-x} = \frac{x}{x-2} + \frac{3}{-(x-2)} = \frac{x}{x-2} - \frac{3}{x-2} = \frac{x-3}{x-2}$$

$$b) \frac{p}{q^2} + \frac{r}{pq} = \frac{p \cdot p}{q^2 \cdot p} + \frac{r \cdot q}{pq \cdot q} = \frac{p^2 + qr}{pq^2}$$

$$c) \frac{4}{y^2 + y} + \frac{3}{y+1} - 2 = \frac{4}{y(y+1)} + \frac{3 \cdot y}{(y+1) \cdot y} - \frac{2 \cdot y(y+1)}{y(y+1)} \\ = \frac{4 + 3y - 2y^2 - 2y}{y(y+1)} = \frac{4 + y - 2y^2}{y^2 + y}$$

$$d) \frac{1}{x} + \frac{1}{x^2 - 1} - \frac{1}{x-1} \\ = \frac{1 \cdot (x+1)(x-1)}{x \cdot (x+1)(x-1)} + \frac{1 \cdot x}{(x^2 - 1) \cdot x} - \frac{1 \cdot x \cdot (x+1)}{(x-1) \cdot x \cdot (x+1)} \\ = \frac{x^2 - 1 + x - x^2 - x}{x(x+1)(x-1)} = \frac{-1}{x(x+1)(x-1)}$$

$$e) \frac{6x^2}{4x^2 - 9} + \frac{6x-3}{4x+6} - 3 \\ = \frac{6x^2 \cdot 2}{(2x+3)(2x-3) \cdot 2} + \frac{(6x-3)(2x-3)}{2(2x+3)(2x-3)} - \frac{3 \cdot 2(2x+3)(2x-3)}{2(2x+3)(2x-3)} \\ = \frac{12x^2 + 12x^2 - 6x - 18x + 9 - 24x^2 + 54}{2(2x+3)(2x-3)} \\ = \frac{-24x + 63}{2(2x+3)(2x-3)} = \frac{-3(8x-21)}{2(2x+3)(2x-3)}$$

$$\begin{aligned}
 \text{f)} \quad & \frac{1}{1-x} - \frac{2}{x-x^2} + \frac{2}{x-x^3} = \frac{1}{1-x} - \frac{2}{x \cdot (1-x)} + \frac{2}{x(1+x)(1-x)} \\
 & = \frac{1 \cdot x(1+x)}{(1-x) \cdot x(1+x)} - \frac{2 \cdot (1+x)}{x \cdot (1-x) \cdot (1+x)} + \frac{2}{x(1+x)(1-x)} \\
 & = \frac{x+x^2-2-2x+2}{x^2-x} = \frac{x(1+x)(1-x)}{x(1+x)(1-x)} \\
 & = \frac{x(x-1)}{x(1+x)(1-x)} = \frac{-x(1-x)}{x(1+x)(1-x)} = \frac{-1}{x+1}
 \end{aligned}$$

$$\begin{aligned}
 \text{g)} \quad & \frac{2}{10a-25} - \frac{4a+10}{20a^2-125} + \frac{1}{2a^2+5a} \\
 & = \frac{2}{5(2a-5)} - \frac{4a+10}{5(2a+5)(2a-5)} + \frac{1}{a(2a+5)} \\
 & = \frac{2a \cdot (2a+5) - (4a+10) \cdot a + 5(2a-5)}{5a(2a+5)(2a-5)} \\
 & = \frac{4a^2+10a-4a^2-10a+10a-25}{5a(2a+5)(2a-5)} \\
 & = \frac{10a-25}{5a(2a+5)(2a-5)} = \frac{5(2a-5)}{5a(2a+5)(2a-5)} \\
 & = \frac{1}{2a^2+5a}
 \end{aligned}$$

OPPGAVE 1.32

$$\text{a)} \quad \frac{c}{3ab} \quad \text{b)} \quad \frac{x+y}{x^2} \quad \text{c)} \quad \frac{a}{a+1} \quad \text{d)} \quad \frac{a+b}{6(x-y)}$$

OPPGAVE 1.33

Utfør divisjonene og gjør svarene så enkle som mulig.

$$\text{a)} \quad \frac{3}{4} : \frac{7}{8} = \frac{3 \cdot 8}{4 \cdot 7} = \frac{3 \cdot 2}{7} = \frac{6}{7}$$

$$\text{b)} \quad \frac{x}{9} : \frac{21}{2x} = \frac{x \cdot 2x}{9 \cdot 21} = \frac{2x^2}{189}$$

$$\text{c)} \quad \frac{z^2}{3} : \frac{z^3}{6} = \frac{z^2 \cdot 6}{3 \cdot z^3} = \frac{2}{z}$$

$$\text{d)} \quad \frac{2x^2-4}{x-1} : \frac{2}{x-1} = \frac{2(x^2-2) \cdot (x-1)}{(x-1) \cdot 2} = x^2-2$$

$$\text{e)} \quad \left(1 + \frac{5}{x}\right) : \left(\frac{x^2+5x}{x^2}\right) = \frac{x+5}{x} \cdot \frac{x^2}{x(x+5)} = 1$$

$$\begin{aligned}
 \text{f)} \quad & \left(\frac{3}{a^2-9} + \frac{2}{a+3}\right) : \left(2a - \frac{2a^2-6a+18}{a+3}\right) \\
 & = \frac{3+2(a-3)}{(a+3)(a-3)} : \frac{2a(a+3)-2a^2+6a-18}{a+3} \\
 & = \frac{2a-3}{(a+3)(a-3)} \cdot \frac{a+3}{2a^2+6a-2a^2+6a-18} \\
 & = \frac{2a-3}{(a+3)(a-3)} \cdot \frac{a+3}{6(2a-3)} = \frac{1}{6(a-3)}
 \end{aligned}$$

OPPGAVE 1.34 Skriv brøkene enklere.

$$\begin{array}{lll} \text{a)} \frac{t+2}{12t} & \text{b)} \frac{17x}{20x+5} & \text{c)} \frac{-(x^2+y^2)}{x^2-y^2} \\ \text{d)} \frac{1}{2a} & \text{e)} \frac{1}{(a-1)(a-2)} & \text{f)} \frac{z^4}{2xy^3} \end{array}$$

OPPGAVE 1.35 Gjør uttrykkene enklere.

$$\begin{array}{l} \text{a)} \sqrt[3]{x^2} \cdot \sqrt[4]{x^3} = x^{\frac{2}{3}} \cdot x^{\frac{3}{4}} = x^{\frac{17}{12}} \\ \text{b)} \left(a^{\frac{1}{2}}\right)^{\frac{2}{3}} = a^{\frac{1}{2} \cdot \frac{2}{3}} = a^{\frac{1}{3}} = \sqrt[3]{a} \\ \text{c)} (\sqrt{8})^{\frac{3}{2}} = (2^3)^{\frac{1}{2} \cdot \frac{3}{2}} = 2^{3 \cdot \frac{1}{2} \cdot \frac{3}{2}} = 2^{\frac{9}{4}} \\ \text{d)} (\sqrt{32})^{\frac{2}{5}} = (2^5)^{\frac{1}{2} \cdot \frac{2}{5}} = 2^{5 \cdot \frac{1}{2} \cdot \frac{2}{5}} = 2 \end{array}$$

OPPGAVE 1.36 a) $10 \cdot 2^{\frac{1}{5}} \approx 11,49$ b) $5 \cdot 2^{\frac{9}{10}} \approx 9,33$ c) $60 \cdot \sqrt{3} \approx 103,92$

d) $125 \cdot 5^{\frac{3}{10}} \approx 202,58$

OPPGAVE 1.37 Gjør uttrykkene enklere.

$$\begin{array}{l} \text{a)} \sqrt[3]{\sqrt{x}} = \left(x^{\frac{1}{3}}\right)^{\frac{1}{2}} = x^{\frac{1}{3} \cdot \frac{1}{2}} = x^{\frac{1}{6}} = \sqrt[6]{x} \\ \text{b)} \sqrt{x} \cdot \sqrt[5]{x^2} = x^{\frac{1}{2}} \cdot x^{\frac{2}{5}} = x^{\frac{1}{2} + \frac{2}{5}} = x^{\frac{9}{10}} \\ \text{c)} \frac{\sqrt{x^3}}{\sqrt[3]{x^2}} = \frac{x^{\frac{3}{2}}}{x^{\frac{2}{3}}} = x^{\frac{3}{2} - \frac{2}{3}} = x^{\frac{5}{6}} \\ \text{d)} \frac{\sqrt[3]{a^2b} \cdot ab}{\sqrt{ab}} = \frac{(a^2b)^{\frac{1}{3}} ab}{(ab)^{\frac{1}{2}}} = a^{\frac{2}{3}} b^{\frac{1}{3}} a b a^{-\frac{1}{2}} b^{-\frac{1}{2}} = a^{\frac{2}{3} + 1 - \frac{1}{2}} b^{\frac{1}{3} + 1 - \frac{1}{2}} \\ = a^{\frac{7}{6}} b^{\frac{5}{6}} \end{array}$$

OPPGAVE 1.38 a) $x = -22$ b) $x = -\frac{9}{5}$ c) $x = -\frac{30}{13}$
 d) $x = \frac{1}{3}$ e) Ingen løsning f) kr 19

OPPGAVE 1.39 Vi skal løse annengradslikningene, merk at de ikke har førstegradsledd (x -ledd)

a) $x^2 - 7 = 0$ $x = \pm\sqrt{7}$

b) $12x^2 - 27 = 0$ $12x^2 = 27$
 $x^2 = \frac{27}{12} = \frac{9}{4}$ $x = \pm\frac{3}{2}$

c) $\frac{1}{3}x^2 = 3$ $x^2 = 9$ $x = \pm 3$

d) $\frac{1}{x^2 - 2x} - \frac{x-5}{x-2} = \frac{5}{x}$ Felles nevner er $x(x-2)$

$$\frac{1 - (x-5) \cdot x - 5 \cdot (x-2)}{x(x-2)} = 0$$

$$1 - x^2 + 5x - 5x + 10 = 0$$

$$-x^2 + 11 = 0$$

$$x^2 = 11$$

$$x = \pm\sqrt{11}$$

OPPGAVE 1.40 a) $x_1 = 0$ og $x_2 = -4$
 b) $x_1 = 0$ og $x_2 = \frac{5}{3}$
 c) $x = 1$

OPPGAVE 1.41 Vi finner løsninger til likningene ved å bruke a-b-c-formelen.

a) $x^2 - 8x + 12 = 0$

$$x = \frac{-(-8) \pm \sqrt{64 - 4 \cdot 1 \cdot 12}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$$

$$x_1 = \frac{8+4}{2} = 6 \quad x_2 = \frac{8-4}{2} = 2$$

b) $x^2 - x - 6 = 0$

$$x = \frac{-(-1) \pm \sqrt{1 - 4 \cdot 1 \cdot (-6)}}{2} = \frac{1 \pm \sqrt{25}}{2} = \frac{1 \pm 5}{2}$$

$$x_1 = \frac{1+5}{2} = 3 \quad x_2 = \frac{1-5}{2} = -2$$

c) $x^2 + 7x + 10 = 0$

$$x = \frac{-7 \pm \sqrt{49 - 4 \cdot 1 \cdot 10}}{2} = \frac{-7 \pm \sqrt{9}}{2} = \frac{-7 \pm 3}{2}$$

$$x_1 = \frac{-7+3}{2} = -2 \quad x_2 = \frac{-7-3}{2} = -5$$

d) $x^2 + 2x - 6 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-6)}}{2} = \frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm \sqrt{4 \cdot 7}}{2}$$

$$x_1 = \frac{-2+2\sqrt{7}}{2} = -1 + \sqrt{7} \quad x_2 = \frac{-2-2\sqrt{7}}{2} = -1 - \sqrt{7}$$

OPPGAVE 1.42 a) $x = \frac{5 \pm \sqrt{97}}{4}$ b) $r_1 = -5$ og $r_2 = -7$

c) $a = \frac{-5 \pm \sqrt{57}}{4}$ d) $p_1 = 8$ og $p_2 = -12$

OPPGAVE 1.43 Vi skal faktorisere uttrykkene. Vi løser de tilsvarende likningene og bruker formelen $ax^2 + bx + c = a(x - x_1)(x - x_2)$

a) $x^2 - 15x + 56 = 0$

$$x = \frac{15 \pm \sqrt{225 - 4 \cdot 56}}{2} = \frac{15 \pm 1}{2} = \frac{8}{7}$$

$$x^2 - 15x + 56 = (x - 8)(x - 7)$$

$$\text{b) } s^2 + 7s + 10 = 0$$

$$s = \frac{(-7) \pm \sqrt{49 - 4 \cdot 1 \cdot 10}}{2} = \frac{-7 \pm \sqrt{9}}{2} = \frac{-7 \pm 3}{2}$$

$$s_1 = \frac{-7 + 3}{2} = -2 \quad s_2 = \frac{-7 - 3}{2} = -5$$

$$s^2 + 7s + 10 = (s - (-2))(s - (-5)) = (s + 2)(s + 5)$$

$$\text{c) } 5A^2 - 4A - 1 = 0$$

$$A = \frac{4 \pm \sqrt{16 - 4 \cdot 5 \cdot (-1)}}{10} = \frac{4 \pm 6}{10}$$

$$A_1 = 1 \quad A_2 = -\frac{1}{5}$$

$$5A^2 - 4A - 1 = 5(A - 1)\left(A + \frac{1}{5}\right)$$

$$\text{d) } 7 - 3y - 4y^2 = 0$$

$$4y^2 + 3y - 7 = 0$$

$$y = \frac{-3 \pm \sqrt{9 - 4 \cdot 4 \cdot (-7)}}{8} = \frac{-3 \pm 11}{8}$$

$$y_1 = 1 \quad y_2 = -\frac{14}{8} = -\frac{7}{4}$$

$$7 - 3y - 4y^2 = -4(y - 1)\left(y + \frac{7}{4}\right) = (1 - y)(4y + 7)$$

OPPGAVE 1.44

$$\text{a) } x_1 = 3 \quad \text{og} \quad x_2 = -3$$

$$\text{b) } x_1 = 0 \quad \text{og} \quad x_2 = -12$$

$$\text{c) } x_1 = 1 \quad \text{og} \quad x_2 = 2$$

$$\text{d) } x_1 = -1 \quad \text{og} \quad x_2 = -15$$

$$\text{e) } x_1 = -4 \quad \text{og} \quad x_2 = -\frac{1}{3}$$

$$\text{f) } x = -\frac{5}{2}$$

$$\text{g) Ingen løsning}$$

OPPGAVE 1.45

To partall som kommer etter hverandre, kan vi skrive som

$$x \quad \text{og} \quad x + 2$$

Da får vi likningen

$$(x)^2 + (x + 2)^2 = 340$$